Solving Temporal Problems using SMT: Strong Controllability

Alessandro Cimatti **Andrea Micheli** Marco Roveri

Embedded Systems Unit, Fondazione Bruno Kessler
Trento, Italy
amicheli@fbk.eu

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Outline

1. The Strong Controllability Problem
2. SMT-based encodings
   - DTPU encodings
   - TCSPU specific encodings
3. Experimental Evaluation
4. Conclusion

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Scheduling for planning applications

The motivating problem

Planning subject to temporal constraints, when the agent cannot control on the actual duration of all the activities.

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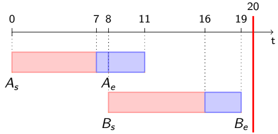
<tr><td>Activities</td><td></td><td></td></tr>
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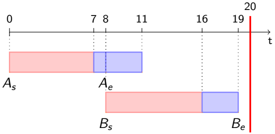
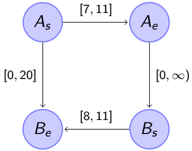
Activities		
TimePoints		

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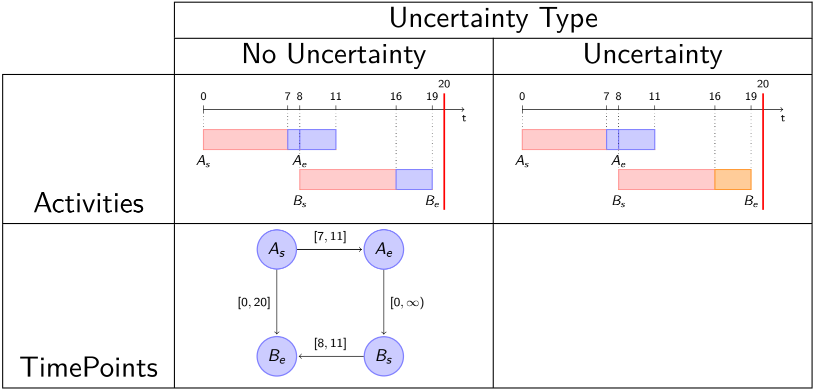

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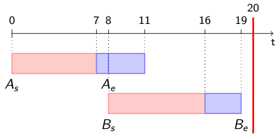
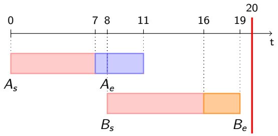
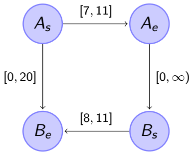
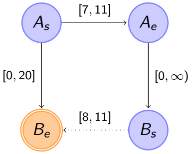
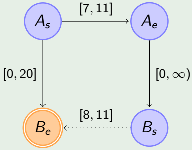
Activities		
TimePoints		

Temporal Problems with Uncertainty

Example



A_s, A_e, B_s are **Controllable Time Points (X_c)**
B_e is an **Uncontrollable Time Point (X_u)**

$\longrightarrow$ represents **Free Constraints (C_f)**
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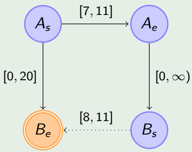

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Taxonomy

Let $\{x_1, \ldots, x_k\} \doteq X_c \cup X_u$.

STPU	TCSPU	DTPU
No disjunctions $(x_i - x_j) \in [l, u]$	Interval disjunctions $(x_i - x_j) \in \bigcup_w [l_w, u_w]$	Arbitrary disjunctions $\bigvee_w ((x_{i_w} - x_{j_w}) \in [l_w, u_w])$

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Intuition

Search for a **Fixed Schedule** that fulfills all free the constraints in every situation.

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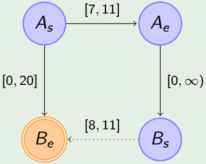

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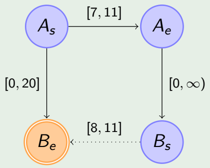

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Definition

A temporal problem with uncertainty is **Strongly Controllable** if

$$\exists \vec{X}_c . \forall \vec{X}_u . (C_c(\vec{X}_c, \vec{X}_u) \rightarrow C_f(\vec{X}_c, \vec{X}_u))$$

where $\vec{X}_c$ and $\vec{X}_u$ are the vectors of controllable and uncontrollable time points respectively, $C_c(\vec{X}_c, \vec{X}_u)$ are the contingent constraints and $C_f(\vec{X}_c, \vec{X}_u)$ are the free constraints.

Contributions

First comprehensive implemented solver for Strong Controllability

- Logic-based framework for Temporal Problems with Uncertainty
- Efficient encodings of Strong Controllability problems in SMT
- Extensive experimental evaluation of the approach

Outline

Satisfiability Modulo Theory (*SMT*)

SMT is the problem of deciding satisfiability of a first-order Boolean combination of theory atoms in a given theory T.

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Theories

Various theories can be used.

In this work:

- *LRA* (*Linear Real Arithmetic*)
- *QF_LRA* (*Quantifier-Free Linear Real Arithmetic*)

Quantifier Elimination in *LRA*

Quantifier Elimination Definition

A theory T has quantifier elimination if for every formula Φ , there exists another formula Φ_{QF} without quantifiers which is *equivalent* to it (modulo the theory T)

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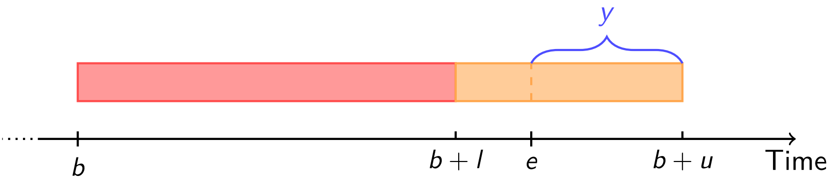
LRA theory admits quantifier elimination, but elimination algorithms are very costly (doubly exponential in the size of the original formula).

Example

$$(\exists x.(x \geq 2y + z) \wedge (x \leq 3z + 5)) \leftrightarrow (2y - 2z - 5 \leq 0)$$

First step: Uncontrollability Isolation

Let $e \in X_u$ and $b \in X_c$.
For every contingent constraint $(e - b) \in [l, u]$, we introduce an offset $y \doteq b + u - e$.



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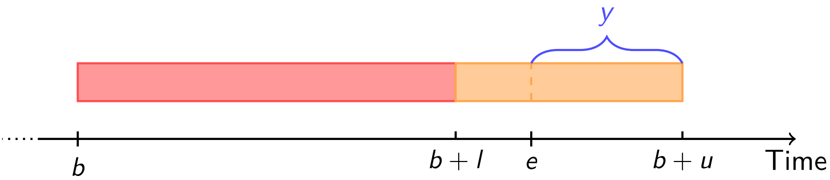
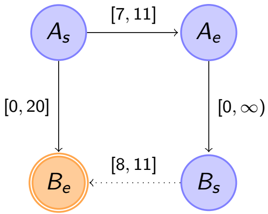


Definition

- Let $\vec{Y}_u$ be the offsets for a given Temporal Problem with Uncertainty
- Let $\Gamma(\vec{Y}_u)$ be the rewritten Contingent Constraints
- Let $\Psi(\vec{X}_c, \vec{Y}_u)$ the rewritten Free Constraints.

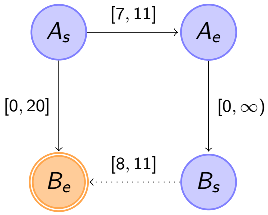
Uncontrollability Isolation: example



Original formulation

$$
\begin{aligned}
&\exists A_s, A_e, B_s.\, \forall B_e. \\
&\quad ((B_e - B_s) \in [8, 11]) \rightarrow (((A_e - A_s) \in [7, 11]) \\
&\qquad\qquad \wedge ((B_e - A_s) \in [0, 20]) \\
&\qquad\qquad \wedge ((B_s - A_e) \in [0, \infty)))
\end{aligned}
$$

Uncontrollability Isolation: example



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Rewritten formulation with Y_{B_e} offset

$$\exists A_s, A_e, B_s. \forall Y_{B_e}.$$
$$(Y_{B_e} \in [0, 3]) \rightarrow (((A_e - A_s) \in [7, 11]) \wedge (((B_s + 11 - Y_{B_e}) - A_s) \in [0, 20]) \wedge ((B_s - A_e) \in [0, \infty)))$$

- $\vec{Y}_u = [Y_{B_e}]$
- $\Gamma(\vec{Y}_u) = (Y_{B_e} \in [0, 3])$
- $\Psi(\vec{X}_c, \vec{Y}_u) = (((A_e - A_s) \in [7, 11]) \wedge \ldots \in [0, \infty)))$

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Direct and Naïve encodings

Direct Encoding

Strong Controllability definition is by itself an encoding in SMT(LRA)

$$\exists \vec{X}_c . \forall \vec{X}_u . (C_c(\vec{X}_c, \vec{X}_u) \rightarrow C_f(\vec{X}_c, \vec{X}_u))$$

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Naïve Encoding

Thanks to uncontrollability isolation, Strong Controllability can be rewritten as follows.

$$\exists \vec{X}_c . \forall \vec{Y}_u . (\Gamma(\vec{Y}_u) \rightarrow \Psi(\vec{X}_c, \vec{Y}_u))$$

Distributed Encoding

Idea: because of the cost of quantifier elimination, many small quantifications can be solved more efficiently than a big single one.

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Starting Point

We assume $\Psi(\vec{X}_c, \vec{Y}_u)$

$$\Psi(\vec{X}_c, \vec{Y}_u) \doteq \bigwedge_h \psi_h(\vec{X}_{c_h}, \vec{Y}_{u_h})$$

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Distributed Encoding

From the Naïve Encoding we can derive a Distributed Encoding, by pushing the quantifications:

$$\exists \vec{X}_c. \bigwedge_h \forall \vec{Y}_{u_h}.(\neg\Gamma(\vec{Y}_u)|_{Y_{u_h}} \vee \psi_h(\vec{X}_{c_h}, \vec{Y}_{u_h}))$$

Eager ∀ Elimination Encoding

Idea: Starting from *Distributed Encoding*, we can eliminate quantifiers during the encoding, producing a *QF_LRA* formula.

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Encoding

Let

$$\psi_h^{\Gamma}(\vec{X}_{c_h}) \doteq \neg\exists \vec{Y}_{u_h}.(\Gamma(\vec{Y}_{u_h})|_{Y_{u_h}} \wedge \neg\psi_h(\vec{X}_{c_h}, \vec{Y}_{u_h}))$$

1. Resolve $\psi_h^{\Gamma}(\vec{X}_{c_h})$ for every clause independently using a quantifier elimination procedure
2. Solve the *QF_LRA* encoding:

$$\exists \vec{X}_c. \bigwedge_h \psi_h^{\Gamma}(\vec{X}_{c_h})$$

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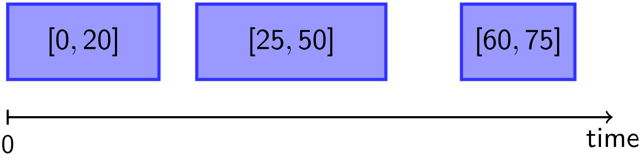
Exploit *TCSPU* structure

Consider a single *TCSPU* constraint:

$B - A \in$



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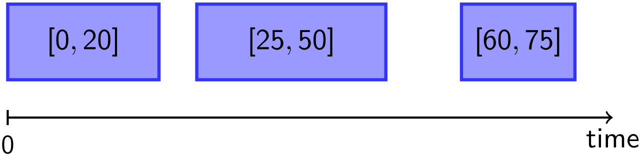

Encoding TCSPU constraints in 2-CNF (Hole Encoding)

$$
\begin{aligned}
&((B - A) > 0) \\
\wedge\ &((B - A) < 20) \vee ((B - A) > 25) \\
\wedge\ &((B - A) < 50) \vee ((B - A) > 60) \\
\wedge\ &((B - A) < 75)
\end{aligned}
$$

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Idea: Exploit Hole Encoding for *TCSPU* to statically resolve quantifiers in the Eager ∀ elimination encoding.

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Hole Encoding gives us a 2-CNF formula. We can enumerate all the possible (8) cases and statically resolve the quantification.

Static quantification *TCSPU*

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Approach

Hole Encoding gives us a 2-CNF formula. We can enumerate all the possible (8) cases and statically resolve the quantification.

Cases

Let $b_i, b_j \in X_c$, $e_i, e_j \in X_u$.
The only possible clauses in the Hole Encoding are in the form:

- $(b_i - b_j) \leq k$
- $(e_i - b_j) \leq k$
- $(b_i - e_j) \leq k$
- $(e_i - e_j) \leq k$
- $(b_i - b_j) \leq k_1 \vee (b_i - b_j) \geq k_2$
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Static quantification *TCSPU* (Example)

Let $b \in X_c$, $e \in X_u$ and let y_e be the offset for e.
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$$C \doteq (b - e) \leq u \vee (b - e) \geq l$$

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In the eager $\forall$ elimination encoding we have

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The formula can be **statically** simplified

$$R \doteq ((l - b + b_e + u_e \leq 0) \vee (l - b + b_e + l_e > 0)) \wedge \\ ((l - b + b_e + l_e < 0) \vee (b - b_e - u - l_e \leq 0))$$

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Whenever a clause matches the structure of C we can derive $\psi_h^{\Gamma}(\vec{X}_{c_h})$ by substituting appropriate values for l, u, b_e, l_e and u_e in R.

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Strong Controllability Results

- Random instance generator
- *SMT* solvers:
  - Z3 (QF_LRA, LRA)
  - MathSAT5 (QF_LRA)
- Quantification techniques:
  - Z3 simplifier
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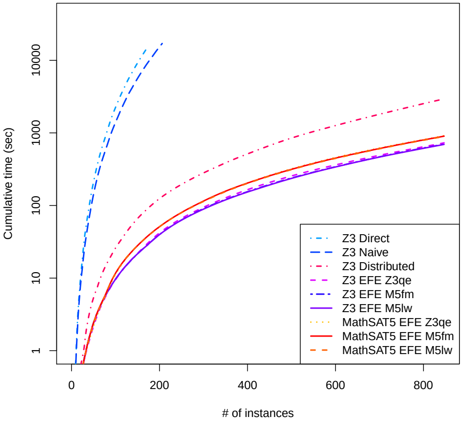
Strong Controllability Results

STPU Results



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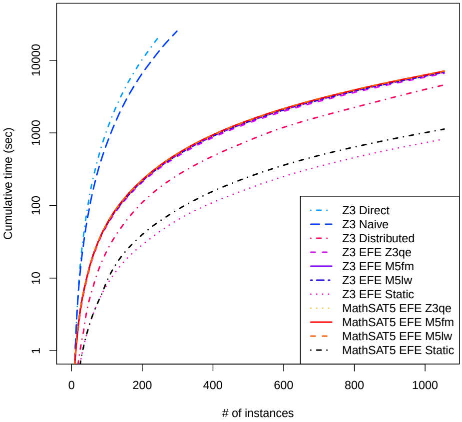
***TCSPU* Results**



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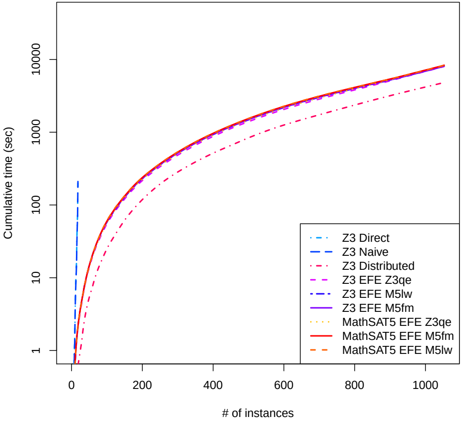
DTPU Results



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Future works

- Dynamic Controllability
- Cost function optimization
- Incrementality

Thanks

Thanks for your attention!

Bibliography

Bart Peintner, Kristen Brent Venable, and Neil Yorke-Smith. Strong controllability of disjunctive temporal problems with uncertainty. In *Principles and Practice of Constraint Programming - CP*, pages 856–863, 2007.

Thierry Vidal and Hélène Fargier. Handling contingency in temporal constraint networks: from consistency to controllabilities. *Journal of Experimental Theoretical Artificial Intelligence*, 11(1):23–45, 1999.

Backup Slides

Related work

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- Strong Controllability of DTPUs has been theoretically tackled in Peintner et al. [2007] using Meta-CSP techniques.
- We developed SMT-based encodings also for Weak Controllability decision problem, and a portfolio of SMT-based algorithms for strategy extraction.

Temporal Problems with Uncertainty

Definition

A Temporal Problem with Uncertainty is a tuple (X_c, X_u, C_c, C_f).

- $X_c \doteq \{b_1, ..., b_n\}$ is the set of *controllable time points*
- $X_u \doteq \{e_1, ..., e_m\}$ is the set of *uncontrollable time points*
- $C_c \doteq \{cc_1, ..., cc_m\}$ is the set of *contingent constraints*
- $C_f \doteq \{cf_1, ..., cf_h\}$ is the set of *free constraints*

$cc_i \doteq (e_i - b_{j_i}) \in [l_i, u_i]$ $\qquad$ $cf_i \doteq \bigvee_{j=1}^{D_i} (x_{i,j} - y_{i,j}) \in [l_{i,j}, u_{i,j}]$

- $j_i \in [1 \ldots n]$
- $l_i, u_i \in \mathbb{R}$
- $l_i \leq u_i$
- $l_{i,j}, u_{i,j} \in \mathbb{R} \cup \{+\infty, -\infty\}$
- $l_{i,j} \leq u_{i,j}$
- D_i is the number of disjuncts for the i-th constraint
- $x_{i,j}, y_{i,j} \in X_c \cup X_u$

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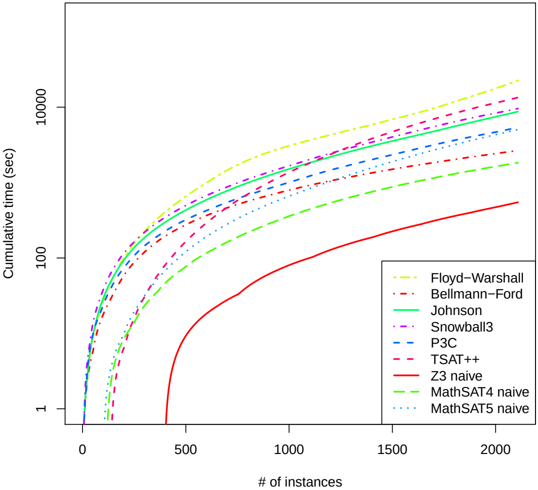
- The *Nature* must fulfill a set of temporal constraints called **Contingent Constraints (C_c)**

Extended Temporal Problem Taxonomy

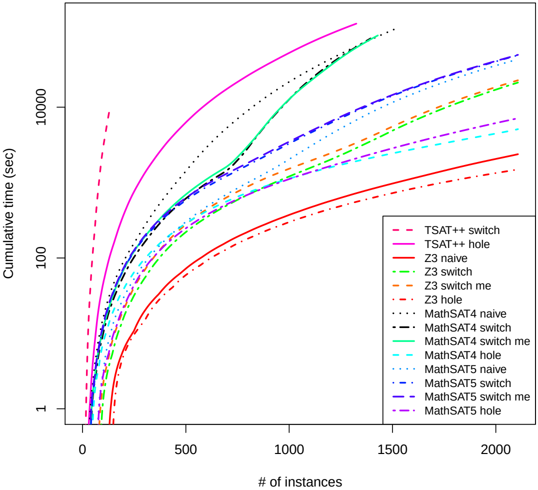
Let $\{x_1, ..., x_k\}$ be the set of all time points of the temporal problem (with uncertainty).

		Uncertainty Type	
		No Uncertainty	Uncertainty
Constraint Type	No disjunctions $(x_i - x_j) \in [l, u]$	STP	STPU
	Interval disjunctions $(x_i - x_j) \in \bigcup_w [l_w, u_w]$	TCSP	TCSPU
	Arbitrary disjunctions $\bigvee_w ((x_{i_w} - x_{j_w}) \in [l_w, u_w])$	DTP	DTPU

Consistency of *STP*



Consistency of *TCSP*



Consistency of *DTP*